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SYSTEMS ASSOCIATES, INC.
444 West Ocean Boulevard
Long Beach, California 90802

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ADVANCED COMMUNICATION SYSTEM TIME DOMAIN MODELING TECHNIQUES STUDY

(ASYSTD)

FINAL PROGRESS REPORT

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Houston, Texas 77058

Contract No. NAS9-11743

SYSTEMS ASSOCIATES

444 West Ocean Boulevard
Long Beach, California 90802
Telephone 213/435-8282

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SECTION 1.0

SUMMARY

This Final Progress Report presents ASYSTD activities dealing with signal to noise ratio and bit error rate measurement, distortion measurement, optimization feasibility, and the definition and systems design implications of mean square error for non-ideal, orthogonally encoded channels. The narrative and supporting technical material follow.

Prepared by:

M. Fashano
Associate ASYSTD Study Manager

Dr. R. M. Gagliardi
Senior Consultant

J. Sullivan
ASYSTD Analysis Task Leader

Approved by:

R. J. Rechter
ASYSTD Study Manager

SECTION 2.0

DESCRIPTIVE NARRATIVE

This progress report deals with the following topics:

1) Description of a procedure for implementing signal to noise ratio measurements using ASYSTD.

2) Description of a procedure for implementing bit error rate measurements using ASYSTD.

3) Signal distortion and criteria which may be implemented using an available ASYSTD post-processing routine.

4) Consideration of the feasibility of system optimization using ASYSTD in conjunction with available iterative optimization.

5) Discussion of Mean Square Error for an orthogonally encoded digital system with a non-ideal channel as it effects both performance and design procedures.

SECTION 3.0

STUDY PROGRAM STATUS

The ASYSTD Study is being concluded approximately according to schedule. This Final Monthly Progress Report covers the remaining ASYSTD analysis tasks as described in Reference ASYSTD/001. Completed ASYSTD program documentation will be presented shortly in the form of a two volume final report.

SECTION 4.0

RECOMMENDATIONS

With the imminent conclusion of the ASYSTD Study it seems appropriate to review suggested enhancements to ASYSTD Program capabilities which might be implemented in future efforts. These potential ASYSTD developments fall into three general categories:

1) ASYSTD Language Enhancements
2) ASYSTD Library Enhancements
3) Advanced Model Development and Implementations

These developments are discussed in the following paragraphs.

4.1 ASYSTD PHASE I LANGUAGE ENHANCEMENTS

4.1.1 Multi-Node Input/Output Models

This task, a major revision of ASYSTD, would result in a completely general topological model definition, and eliminate the use of "TAPS" for multiple input/output devices as currently required.

4.1.2 Enhance Expression Processing

This task will provide the capability of using node names as variables in any expression, and scanning any expression for proper form.

4.1.3 Checkpoint Feature

This task will provide automatic, user controlled checkpointing keyed to the computer system clock and system run controls.

4.1.4 Conditional Termination

This task will provide for automatic termination of any problem iteration, or the run itself, based upon user supplied "IF" statements. This feature would allow a user to monitor any system variable to determine its relation to some predetermined quantity, and terminate the run if the condition is satisfied.

4.1.5 Incorporate Table Definitions

This task will provide the capability of defining a table of arbitrary size and dimension, consisting of constants, much the same as in a FORTRAN dimension statement.

4.1.6 SAVE Feature

This task will implement a SAVE command for use in saving any data written on scratch storage during the simulation. This command would be processed much the same as the PLOT command.

4.1.7 NO-SORT Feature

This task will circumvent the sorting process carried out by ASYSTD. This feature would allow the user to define the sequence in which the expressions will be evaluated. Intermixing of FORTRAN executable statements will also be provided.

4.1.8 Enhance the ASYSTD Sort (COMPIL) Technique

This task will enhance the current ASYSTD expression sorting technique which determines the sequence of operations in the simulation. In addition, provide an immediate operand node so as to cause immediate evaluation of an expression independent of the sorting process.

4.1.9 Program Output Formatting

This task will provide the user with the necessary controls to specify page numbering, titling, and time/dating all output from Phase II of ASYSTD.

4.1.10 Cross-Reference Output

This task will allow the user to request cross reference information as to node name, tap name, etc., versus storage location in the "V" pool. This feature is particularly useful in debugging user written FORTRAN models when using a Phase II core dump.

4.1.11 Automatic Specification of Phase II Core Storage

This task will result in the automatic sizing of the Phase II execution of the simulation relative to required core. This is particularly important in the EXEC 8 timesharing environment. Expansion of the library directory would be necessary to incorporate this feature.

4.1.12 Automatic Library Directory Updating

This task will provide the capability to automatically update the library directory when new models are introduced.

4.1.13 Use of Both Real and Complex Models

This task will provide the capability for defining both real and complex (RF Translated) models, and will minimize the computational and storage overhead exhibited when all models are assumed complex (currently, real models are complex with zero imaginary quantities). Implementation of this task would require expansion of the library directory.

4.2 ASYSTD PHASE II LIBRARY CODE ENHANCEMENTS

4.2.1 Model Debug Output

This task will incorporate canned coding in every ASYSTD model to facilitate debugging. Output specifying the key parameters would be printed at the entrance and exit of each model. The debug output would be controlled from a DEFAULT ASYSTD command.

4.2.2 Advanced Modeling Aids

This task would provide various FORTRAN procedures to the user for interfacing with ASYSTD key parameters and storage areas. In addition, various utility routines could be provided to perform such operations as interpolation, extrapolation, sealing, etc., which are normally available with any system such as FORTRAN libraries.

4.3 ADVANCED MODEL DEVELOPMENT AND IMPLEMENTATION

4.3.1 Digital System Design Using Orthogonal Transform Source Encoding

There has been recent interest in using orthogonal transform source encoding of data time samples at the transmitter to combat quantization noise in digital communications. Early work by Comsat Corporation was devoted to determining the best orthogonal encoding procedure for transmitting digitized voice over ideal channels. More recently, Systems Associates, Inc. has extended these results by considering the effect of channel errors during transmission of the quantized encoded coordinates. This leads to an overall mean square reconstructed error that includes both the quantization and channel effects. When this mean squared error (MSE) can be written in closed form, the overall communication link can theoretically be optimized over both encoding procedures (number bits per quantized word) and transmission parameters (channel bit signal power). Unfortunately, the MSE is generally difficult to compute exactly, and has been approximated only after certain simplifying assumptions. For these special cases, however, it can be shown that the optimal design leads to parameter values different from that generated using the ideal channel (Comsat results), and an intrinsic relation exists between encoding design and channel design such that one cannot be designed without consideration of the other.

4.3.1.1 Comparative Evaluation of Orthogonal Transform Models - In future work further effort will be devoted to determining the optimal design of the overall system, taking into account the orthogonal transform encoding scheme. The primary objective of the study will be an assessment of the various orthogonal encoding schemes and their capabilities, when operating in a practical system with non-ideal channels.

4.3.1.2 Implementation of Orthogonal Transform Methods in ASYSTD - The above analytical studies will be interfaced with the ASYSTD Simulation Program, so that real time channel parameters can be inputted directly. In this way, MSE can be computed directly in terms of a practical operating system. Furthermore, MSE can possibly be computed, by computer programming, for more complicated digital formats such as block-coding, error-corrected encoding, and convolutional coding.

4.3.1.3 Channel Bit and Word Error Probability Measurement - Since ultimate performance can be inherently linked to channel bit and word error probabilities, a requirement exists for measuring these parameters in system simulation. Studies by Systems Associates, Inc. have presented and explored various methods for obtaining these error rate measurements in real time. These methods will be further studied in terms of practicality, complexity, and computer time, and the most promising of these methods will be implemented.

4.3.2 ASYSTD General Device Modeling

4.3.2.1 Extension of Existing General Device Model - It is of interest to determine the ultimate capabilities of the present complex curve fit general device model in various applications of interest. One obvious limitation is a present constraint on the number of input spectrum data points. For some transfer functions to be approximated, this limited number of data points may not provide an adequate description. There is every confidence that this problem can be overcome. The present debilitating constraint limiting the number of response data points describing the transfer function to be modeled will then be eliminated.

4.3.2.2 General Device Model Determination from Topological Circuit Description - A complementary approach to the general device modeling problem, useful when the device description is in the form of a circuit schematic, involves the use of an existing program, NASAP. NASAP (Network Analyses for Systems Applications Program), developed under the auspices of NASA, Electronics Research Center, determines the transfer function of a device from a straightforward user-language description of the device equivalent circuit. NASAP requires a device topological description input which is limited to passive and linear elements, but is otherwise completely general. With minor modifications to the existing general device model, the NASAP generated transfer function poles and zeros could be input directly as an alternative option to their internal calculation from device response data. It is felt that this option would provide a useful added flexibility to existing ASYSTD general device modeling capabilities.

4.3.3 Extensions of ASYSTD Propagation and Multipath Models

The existing ASYSTD propagation and multipath models could be enhanced by the following further developments.

4.3.3.1 Atmospheric Propagation Model - Implement a more precise atmospheric propagation model, using current weather data and considering climatology of specific earth stations.

4.3.3.2 Atmospheric Noise Model - Extend the atmospheric noise model to include the contribution of galactic sources, solar noise (pointing constraints result, actually), and noise due to the intercepted earth disc.

4.3.3.3 Multipath Model - Implement a more faithful multipath model, accounting for the more general diffuse reflectivity case, a spherical earth model, and perhaps two secondary rays (for the multiple spacecraft case).

4.3.4 System Performance Measuring Capability

4.3.4.1 Implementation of Rate Distortion Evaluation of Orthogonal Transform Performance - Rate distortion theory is applicable to quantization processes which occur in the course of implementating digital orthonal transformation of analog signals. It would be desirable to establish rate distortion criteria appropriate to the evaluation of each of the orthogonal transform types (Haar, Hadamard, Karhunen-Loeve) considered in the COMSAT orthogonal transform study (NAS9-11240) as applied to voice data and pictorial data. The end result of this implementation will be an internal ASYSTD function for use with each of the orthogonal functions.

SECTION 5.0

TECHNICAL DISCUSSION

5.1 ORTHOGONAL TRANSFORM STUDY

In a previous report, Reference [1], a digital communication system utilizing orthogonal encoding at the transmitter, and operating over a non-ideal channel, was investigated. The orthogonal source encoding was used to improve the quantization error by allowing an optimal digital symbol allocation. The non-ideal channel, however, caused errors to occur in symbol transmission, and produced an added component to the overall system accuracy. This added error effect can sometimes negate the advantage gained in quantization by transmitter encoding. In this report, overall mean squared error (MSE), due to both quantization and non-ideal transmission, is further examined for performance and design procedures.

5.1.1 Mean Squared Error for Non-Ideal Channels

The MSE in an orthogonal encoded digital system using digital transmission over a noisy channel was computed in Reference [2] under the following assumptions:

1) The source message was time sampled and encoded via an orthogonal transformation into transform coordinates. These coordinates were then quantized into digital sequence for transmission. The transforms considered were the fast Fourier (FF), fast Hadamard (FH), Haar (H), and Karhunen-Loeve (KL) transformations.

2) A logarithmically compressed-uniform quantizer was used - this assumption made the coordinate quantization error directly proportional to the coordinate variance,

and inversely proportional to the number of quantization levels devoted that coordinate. The coordinate variances were determined experimentally by processing typical voice waveforms with the candidate transforms. The number of quantization levels devoted to a given coordinate was treated as an unknown parameter to be eventually optimized.

3) Uncorrelated coordinate quantization errors was assumed. This allows the total MSE in transmitting N coordinates to be simply the sum of the individual coordinate mean square quantization errors. In general, this assumption requires that the orthogonal transformation used to generate the coordinates from the time samples be the K-L expansion of the source data. If correlation exists among the coordinates, the individual coordinate variances no longer give a true picture of the energy distribution.

4) The channel used bit by bit PCM transmission, with either coherent or non-coherent signalling, and the bit error probability P_b was such that only a single bit error occurred during transmission of a given quantization word. Although other types of data channels can be considered, the above assumption led to closed form expressions for MSE.

Under the above assumptions the total MSE in the recovered data waveform (after sampling, quantizing, orthogonal encoding, transmission, and decoding) is then

$$MSE = \sum_{i=1}^{N} \left[\frac{\sigma_i^2}{4^{n_i}} + P_{bi} \left(\frac{\sigma_i^2}{4^{n_i}} \right) \left(\frac{4^{n_i} - 1}{3} \right) \right] \qquad (1)$$

where

$$\begin{aligned} N &= \text{Number coordinate samples per frame} \\ \sigma_i^2 &= \text{Variance of } i^{th} \text{ coordinate} \\ n_i &= \text{Number bits per } i^{th} \text{ quantization level} \\ P_{bi} &= \text{Bit error probability for each bit of the } i^{th} \\ & \quad \text{quantization word} \end{aligned} \tag{2}$$

The first term is the contribution due to quantization, while the second is the added effect due to a non-ideal channel; i. e., inaccurate transmission of the PCM information bits during each quantization word. The total number of bits that can be transmitted during a frame of data T sec long depends upon the allowable system bit rate. In particular, the parameter set $\{n_i\}$ must satisfy the requirement

$$\sum_{i=1}^{N} n_i = RT \tag{3}$$

where R is the channel bit rate in bits/sec. In practical systems the bit rate R is controlled by the system bandwidth, so that (2) appears as a constraint condition on (1). In earlier work the performance of the various orthogonal transformations was evaluated by first specifying the bit rate R and frame time T. Then the optimal allocation of the RT bits over the N coordinates was determined so as to minimize the MSE due to quantization. That is, MSE in (1) with $P_{bi} = 0$ was minimized by selection of the $\{n_i\}$ subject to the constraint of (2). The resulting quantization MSE was then normalized by the message energy

$$E_m = \sum_{i=1}^{N} \sigma_i^2 \tag{4}$$

and its reciprocal called the quantized signal to noise ratio:

$$(SNR)_Q = \frac{E_m}{\sum_{i=1}^{N}\left(\frac{\sigma_i^2}{4^{n_i}}\right)} \tag{5}$$

The above represents the ratio of available signal energy to the MSE for an ideal channel. When the set $\{n_i\}$ are chosen to maximize the denominator, the $(SNR)_Q$ is maximized. Since the coordinate variances $\left\{\sigma_i^2\right\}$ depends upon the transformation used, the resulting maximum value of $(SNR)_Q$ will be different for each transformation.

When the effect of a non-ideal channel is included, $P_{bi} \neq 0$, the resulting SNR has a denominator given by (1), rather than that given in (4). Hence,

$$SNR = \frac{E_m}{\sum\left[\left(\frac{\sigma_i^2}{4^{n_i}}\right) + P_{bi}\left(\frac{\sigma_i^2}{4^{n_i}}\right)\left(\frac{4^{n_i} - 1}{3}\right)\right]} \tag{6}$$

$$= \frac{(SNR)_Q}{1 + D}$$

where

$$D = \frac{\sum_{i}^{N} P_{bi}\left(\frac{\sigma_i^2}{4^{n_i}}\right)\left(\frac{4^{n_i} - 1}{3}\right)}{\sum_{i}^{N}\left(\frac{\sigma_i^2}{4^{n_i}}\right)} \tag{7}$$

Since D is greater than one, the effect of channel errors clearly is to degrade the overall SNR, as may be expected. A quantitative examination of this effect can be made by considering the case where bit error probability is identical for every word; i.e., $P_{bi} = P_b$. Then (6) reduces to

$$D = \left(\frac{P_b}{3}\right)\left[(SNR)_Q - 1\right] \tag{8}$$

The resulting SNR in (5) is then plotted in Figure 5-1 as a function of the bit error probability P_b. For low error probabilities, the channel has little effect, and SNR = $(SNR)_Q$ since the MSE is due entirely to quantization error. As higher channel error probabilities occur, however, the SNR exhibits a rapid degradation over the $(SNR)_Q$. In essence, a SNR break point appears at approximately

$$P_b \approx \frac{3}{(SNR)_Q - 1} \approx \frac{3}{(SNR)_Q} \tag{9}$$

Thus, the higher the design $(SNR)_Q$, the lower the error probability at which the channel becomes significant. This means encoding schemes that offer the largest quantization improvement are most susceptible to channel errors.

5.1.2 System Design with Non-Ideal Channels

The MSE in (1) (and therefore the SNR in (5)) represents a measure of the system accuracy in a digital communication system, under the assumptions described in Section 5.1.2. Since the parameter N depends upon the source sampling rate, and the coordinate variances depend explicitly upon the orthogonal encoding scheme used, the system design involves only the control of the word length parameter set $\{n_i\}$ and the channel bit error probability set $\{P_{bi}\}$. The latter set depends upon the manner in which the PCM bits are transmitted over the channel and on the detection scheme used. For example, the error probability for two common modes of transmission are given by

$$P_{bi} = \int_{\gamma_i}^{\infty} e^{-t^2/2}\, dt \qquad \text{PSK, coherent detection} \tag{10a}$$

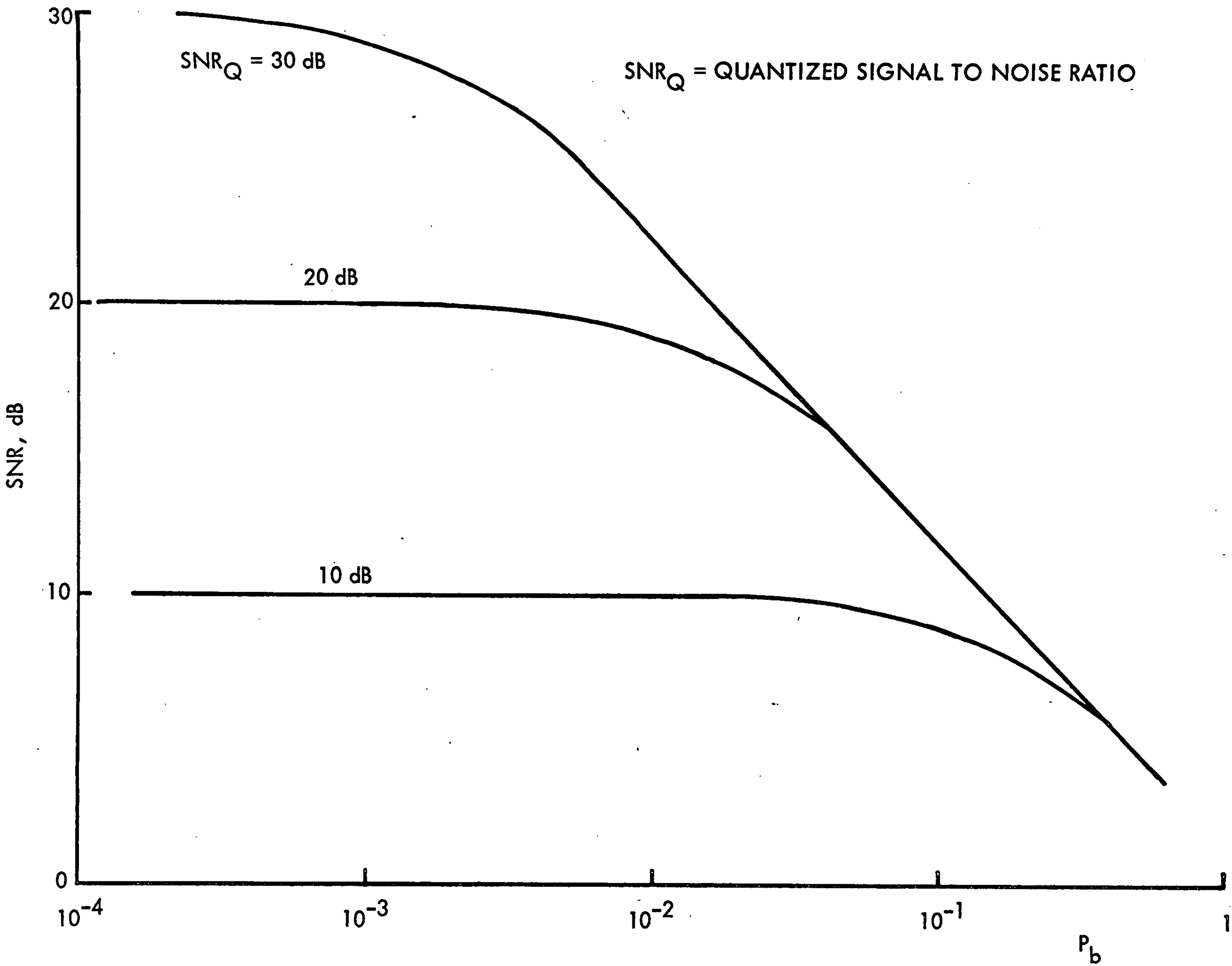


Figure 5-1. SNR vs. Uniform Bit Error Probability ($P_{bi} = P_b$)

$$P_{bi} = \frac{1}{2} \exp\left[\frac{-\gamma_i^2}{2}\right] \qquad \text{FSK, non-coherent detection} \qquad (10b)$$

where γ_i^2 is the received signal energy to noise spectral level of a data bit during the i^{th} word; i.e., during each of the n_i bits that comprise the i^{th} word. Since γ_i^2 is proportional to the average power during the i^{th} word, the sum

$$\sum_{i=1}^{N} \gamma_i^2 \triangleq P \qquad (11)$$

is therefore proportional to the total power available during the transmission of a particular data frame. Hence, (10) appears as a constraint on the total power determining the error probabilities in (9).

The system designer is therefore faced with the problem of maintaining high system accuracy (minimizing MSE) by properly selecting the $\{n_i\}$ and $\{P_{bi}\}$, subject to the constraints of (2) and (10). The optimal system therefore requires joint minimization of MSE over the above parameter sets. This corresponds to the following basic problem:

$$\underset{\{n_i\},\ \{\gamma_i\}}{\text{minimize}} \text{ [MSE]} \qquad (12)$$

subject to the conditions:

$$\sum_{i}^{N} \gamma_i = P, \quad \sum_{i=1}^{N} n_i = RT, \quad n_i, \ \gamma_i \geq 0, \quad n_i = \text{integer} \qquad (13)$$

Since γ_i is a continuous parameter, Calculus of Variations can be used to carry out its minimization. In particular,

$$\frac{\partial}{\partial \gamma_i}\left[\text{MSE} + \lambda \sum \gamma_i^2\right] = 0$$

$$= g(i)\left(\frac{\partial P_{bi}}{\partial \gamma_i}\right) + 2\lambda\gamma_i \tag{14}$$

where

$$g(i) = \frac{\sigma_i^2}{3}\left[1 - \frac{1}{4^{n_i}}\right] \tag{15}$$

and λ is the Lagrange multiplier. The solution depends upon the set $\{n_i\}$, illustrating the joint optimization (simultaneous solution) that must be performed. In addition, the solution will depend upon the type of digital signalling used. From (9),

$$\frac{\partial P_{bi}}{\partial \gamma_i} = \frac{-e^{-\gamma_i^2/2}}{\sqrt{2\pi}} \quad , \text{ PSK} \tag{16a}$$

$$\frac{\partial P_{bi}}{\partial \gamma_i} = \frac{-\gamma_i}{2}\, e^{-\gamma_i^2/2} \quad , \text{ FSK} \tag{16b}$$

and (13) requires solution of either of the equations:

$$e^{-\gamma_i^2/2} = \frac{\lambda}{g(i)}\,\gamma_i \quad , \text{ PSK} \tag{17a}$$

or

$$e^{-\gamma_i^2/2} = \frac{\lambda}{g(i)} \quad , \text{ FSK} \tag{17b}$$

where λ has absorbed all constants. The second simultaneous equation is obtained through minimization of MSE with respect to the integer set $\{n_i\}$, the solution involving a dynamic allocation process under a linear constraint. Specifically, the equation to be solved is

$$f_j(x) = \min_{0 \le n_j \le x} \left[h_j(n_j) + f_{j-1}(x - n_j)\right] \tag{18}$$

for j = 2, 3, . . . N, with x = 1, 2, . . . RT, and

$$f_1(y) = h_1(y) \tag{19a}$$

$$h_i(y) = \frac{\sigma_i^2}{4^y}(1 - P_{bi}/3) \tag{19b}$$

The recurrence relation in (17) yields a systematic method for obtaining the sequence $\{n_i\}$ inductively. Equations (17) and (16) must be solved simultaneously for $\{n_i\}$ and $\{\gamma_i\}$. It is interesting to note that if the P_{bi} is fixed, the solution in (17) is identical to that maximizing $(SNR)_Q$ (the ideal channel solution) with σ_i^2 replaced by $\sigma_i^2(1 - P_{bi}/3)$.

An approximation to the true solution can be obtained by assuming $4^{n_i} \gg 1$, so that in (14)

$$g(i) \approx \frac{\sigma_i^2}{3} \tag{20}$$

and no longer depends upon the $\{n_i\}$. The original joint minimization problem now "uncouples", in the sense it may be solved in two separate parts. Each γ_i is then the solution to either equation in (16). Specifically, we have

$$e^{-\gamma_i^2/2} = \left(\frac{\lambda}{\sigma_i^2}\right)\gamma_i \quad , \text{ PSK} \tag{21a}$$

or

$$e^{-\gamma_i^2/2} = \frac{\lambda}{\sigma_i^2} \quad , \text{ FSK} \tag{21b}$$

Both Equations (19) have a finite, positive, and unique solution. The relative solution in (19a) is shown in Figure 5-2, as a function of relative σ_i^2. (The term relative is used here to imply that adjustment for the constraint condition has not yet been included.) To use this curve the smallest of all σ_i^2 is taken as one, and all others are taken relative to it. The corresponding relative γ_i is read from the curve for each i, leading to a set of relative channel powers $\{\gamma_i\}$. The absolute value of the $\{\gamma_i\}$ satisfying (10) can be determined by multiplying each relative γ_i^2 by

$$P \Big/ \sum_{i=1}^{N} \tag{22}$$

(relative γ_i). It is noted that essentially a linear relation exists between the relative parameters.

The solution to (19b) can be written directly as

$$\gamma_i^2 = 2\ell n\left(\sigma_i^2\right) + 2\ell n\lambda \quad , \text{ FSK} \tag{23}$$

with λ satisfying the constraint. It is seen that in either case γ_i is directly related σ_i^2. This means that for maximum SNR, the power associated with each bit of the i^{th} word will depend upon the variance of that coordinate. In particular, the more important coordinates will have their bits transmitted with more power than less important coordinates. The proper distribution of this power is given by Figure 5-2 or Equation (20) for each i, with λ determined by the constraint in (12). With $\{\gamma_i\}$ selected in this way, (17) can now be solved by dynamic programming for the $\{n_i\}$.

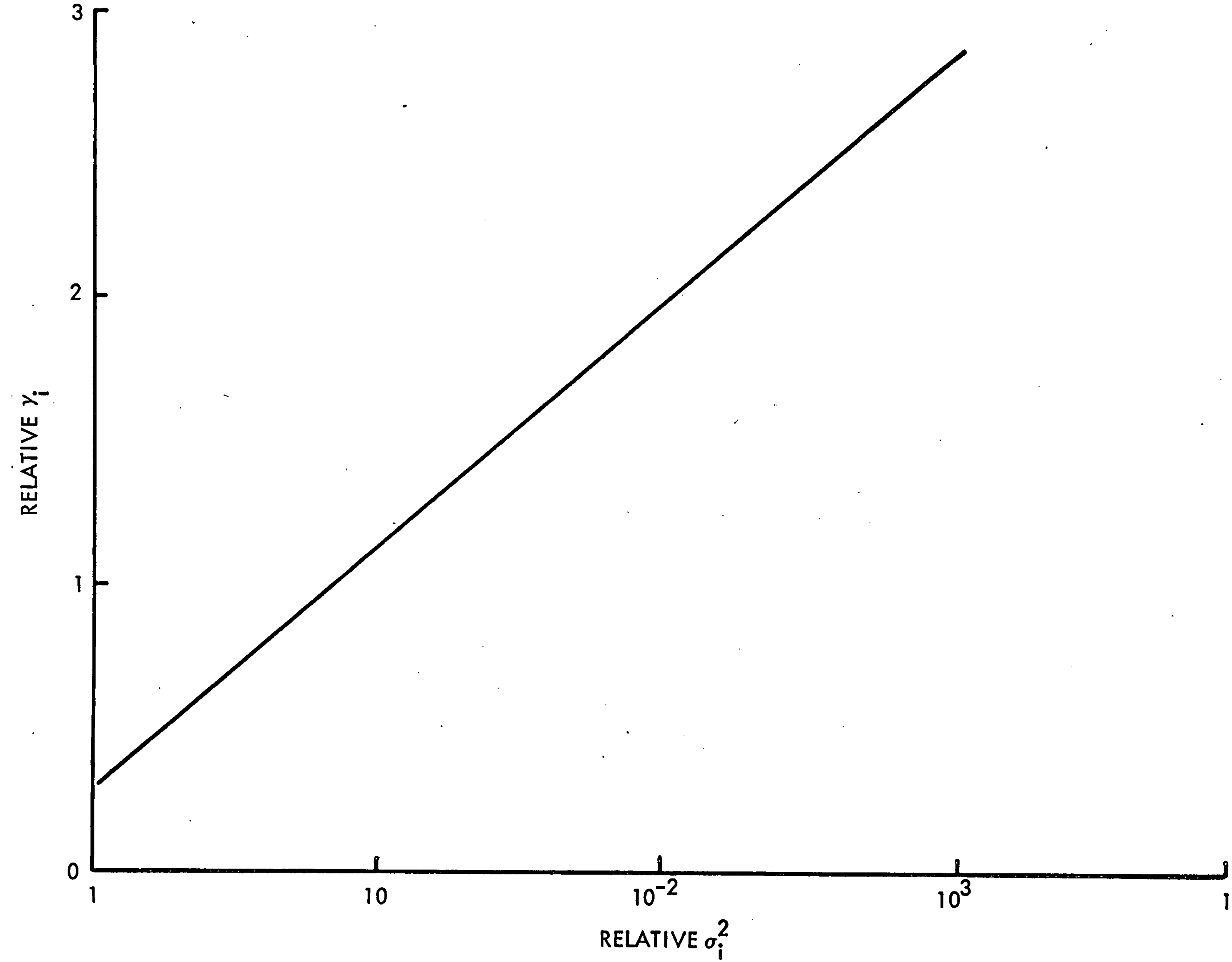


Figure 5-2. Relative γ_i vs. Relative σ_i^2

The dynamic allocation problem essentially requires a computer search over possible integer values. Some simplification can be obtained by accepting an approximate solution for the $\{n_i\}$. One simple way is to omit the constraint that each n_i must be an integer. This allows a MSE minimization over a continuous variable which can be handled explicitly by Calculus of Variations. If the minimum over the $\{n_i\}$ is fairly shallow, then the continuous solution to the $\{n_i\}$ can be "rounded off" to the nearest lower integer. (Note that if the constraint in (12) is satisfied exactly with continuous n_i, it becomes an upper bound when rounding off in this way.) By formal procedures,

$$\frac{\partial}{\partial n_i}\left[\mathrm{MSE} + \lambda \sum n_i\right] = 0$$
$$= -\frac{\sigma_i^2\left[1 - \frac{P_i}{3}\right]}{4^{n_i}} + \lambda \tag{24}$$

from which it follows that

$$n_i = C_1 \log_4\left[C_2 \sigma_i^2\left(1 - \frac{P_{bi}}{3}\right)\right] \tag{25}$$

The constant C_1 and C_2 that satisfies (12) can easily be determined. The implication of (22) is that the word size n_i must be varied over the coordinates, according to the coordinate variances, just as for the ideal channel. The variances, however, are modified by the bit error probability for that word, the latter determined by the solutions from (19).

When the solution in (22) is substituted into (1), the resulting MSE takes the form

$$\mathrm{MSE}\Big|_{\min} = \frac{N}{4^{RC_1}} + \sum_{i=1}^{N} \frac{P_{bi}\sigma_i^2}{3} \tag{26}$$

where

$$C_1 = T \Bigg/ \sum_{i=1}^{N} \log_4\left[C_2 \sigma_i^2 (1 - P_{bi}/3)\right] \tag{27}$$

and

$$C_2 = 1 \Big/ \min_i \left[\sigma_i^2 (1 - P_{bi}/3)\right] \tag{28}$$

Equation (23) has been obtained with the integer constraint on n_i omitted, and therefore represents a true lower bound to MSE (that is, reinsertion of this constraint can only lead to a larger MSE). Since P_{bi} depends only upon σ_i^2 [see (9), (18), and (19)], the second term in (23) represents a fixed MSE dependent only upon the type of transformation used. The first term depends on the bit rate R, and the resulting MSE decreases exponentially as R increases, even with non-ideal channels.

5.1.3 Improvement Using Channel Word Error Correction

The previous results have been based upon a key assumption that only a single bit error could occur during the transmission of any word. Such operation immediately suggests the use of word error correction at the receiver to improve performance. This can be achieved by sacrificing data bits and using them as parity check bits during a word transmission, allowing bit error identification and correction after bit by bit decoding. If only a single bit error is assumed to occur in a given word, then only one data bit need be sacrificed to identify word errors. This means one less data bit is available for the word, but the channel error effect is eliminated. Thus, for each i in (1), the system designer can operate with the component of the MSE shown, or he may use single error correction during the word transmission, forcing $P_{bi} = 0$ and eliminating one data bit. In the latter case the contribution to the MSE is due entirely to quantization error, and the i^{th} coordinate has an error $\sigma_i^2/4^{n_i-1}$. It is clear that the overall MSE will be improved (decreased) if

$$\frac{\sigma_i^2}{4^{n_i-1}} < \frac{\sigma_i^2}{4^{n_i}} + P_{bi}\left(\frac{\sigma_i^2}{4^{n_i}}\right)\left(\frac{4^{n_i} - 1}{3}\right) \tag{29}$$

or if

$$\frac{9}{4^{n_i} - 1} < P_{bi} \tag{30}$$

Hence, if the bit error probability during the transmission on any word satisfies the above condition, then system improvement can be achieved by using error correction for that word. Since n_i is proportional to σ_i^2 it is clear that if the j^{th} word is corrected then all words more important (having larger coordinate variances) than the j^{th} word should also be bit error corrected.

5.2 SNR MEASUREMENTS USING ASYSTD SIMULATION

Signal-to-Noise Ratio (SNR) can be determined within an ASYSTD run. via use of ASYSTD models and/or mathematical expressions within the simulation topology - much the same as a meter would be connected to monitor a system node. There are two distinct cases which may appear in a simulation:

1) For a linear system Signal (S) and Noise (N) may be validly separated and measured independently. (Signal power is usually known a priori.)

2) For a non-linear system Signal and Noise cannot be validly separated and must be measured as S+N (even though signal power may be known a priori).

Case 1 requires a single run of the simulation since the signal power is known a priori, or can be independently measured in the same run. Use of a power meter is necessary for the measurement of signal or noise. A power meter may be defined as:

$$P_{(T)} = \frac{1}{T} \int_0^T V^2(t)dt \tag{31}$$

and modeled in ASYSTD as:

```
MODEL:  POWER METER
INPUT  < $*$/TIME > N1                        (32)
N1 <INTEGRATE > OUTPUT
END
```

The measurement of SNR for Case 1 can then be made utilizing an ASYSTD expression of the following form:

```
IN < S/$ >  OUT                               (33)
```

where the input node IN is the output of the power meter measuring the noise power at a given node; S is the a priori signal power; and output node OUT is the ratio of signal-to-noise.

An alternate is to generate an SNR meter such as:

```
MODEL:  SNR METER 1, S
INPUT  < POWER METER >  N1                    (34)
N1 < S/$  >OUTPUT
OUTPUT <10.*ALOG10($)> DUM     'TAP1
END
```

The above model is an SNR meter for Case 1, the signal level being provided when the model is referenced.

Case 2 is similar to Case 1 when the signal power is known a priori, and SNR can be measured with the following model:

```
MODEL:  SNR METER 2, S
INPUT  <POWER METER> N1
N1  <1./($/S-1.)>  OUTPUT                     (35)
OUTPUT <10.*ALOG10($)> DUM     'TAP1
END
```

Note that TAP1 is the SNR in DB.

When the signal power, in addition to the noise, must be measured the use of the "VARY" command within ASYSTD is used and two runs are required. In this case, the signal power remains on during both runs. The noise power, however, would be off during the first run and on during the second. This is accomplished by the following statement:

VARY ETA : 0., 10.E-3, 10.E-3 (36)

The spectral density (ETA) is set to zero for the first pass (X_1 - signal only) and 10.E-3 for the final pass (X_2 signal + noise). The SNR may then be computed as follows since:

$$\frac{X_2}{X_1} = \frac{S+N}{S} = 1 + \frac{N}{S} \tag{37a}$$

$$SNR = \frac{1}{\frac{X_2}{X_1} - 1.} \tag{37b}$$

The major problem in measuring SNR occurs with a non-linear system where the total input power determines the output signal characteristics - e.g., a limiter in which the suppression is a function of the total input power (S+N). If S remains constant for both signal and signal and noise measurement, the device characteristic changes, possibly invalidating the measurement. The limiter is a simplistic case in that its characteristics are well known and can be compensated for. The problems arise with devices having unknown characteristics. An alternate approach to the SNR measurement is to relate a quantity such as Bit Error Rate or Signal Distortion to SNR, which can easily be accomplished at higher simulation costs.

In any event, the measurement of SNR requires little effort by the user; however, the interpretation of the measurement should be viewed in light of the system characteristic being investigated.

5.3 BIT ERROR RATE MEASUREMENTS USING ASYSTD SIMULATION

Measurement of bit error probability P_b in an operating or simulated digital system was discussed in the Fourth Monthly Progress Report. Measurement of P_b (equivalent to BER = $P_b \cdot$ Bit Rate) may be determined by various alternative methods:

1) Error counting methods
2) Sampler distribution analysis (SDA)
3) Extreme value analysis (EVA)
4) Baseband SNR estimation

Each of these techniques makes use of a different relationship between bit error probability P_b and some observable system quantity. For the error counting method, the observable is the number of sample values above threshold. For SDA, the observables are the collection of all sample values. For EVA the observables are the largest of a set of sample values. Finally, in SNR estimation, the observables are samples of the analog waveform prior to decoding and sampling.

In a typical data system, the received data bits are first processed, then integrated over the bit period, and finally sampled for threshold comparison. The processing may be coherent (matched filtering) or non-coherent (envelope detected). The bit sample value is compared to a threshold level T, and a binary one or zero is decided if above or below T. In the following discussion, we will be concerned with antipodal coherent signaling. In this case, the threshold is at T = 0 and the mean of the sample value is either positive or negative, depending on the transmitted bit. X_i is defined as the sample value of the ith bit when a zero is sent and $-X_i$ as the sample when a one is sent. The ith bit is correctly decoded if $X_i < T$ and incorrectly if $X_i \geq T$. Hence, the bit error probability for a sample value X is then equivalent to:

$$P_b = P\left[X \geq T\right] \tag{38}$$

The BER estimation problem is then to determine this P_b given the set of sample values, $\{X_i\}$. Implementation of two methods most appropriate to the ASYSTD simulation procedure are discussed below.

Direct Error Counting

Conceptually, the most straightforward technique is simply counting the number of sample values which exceed the specified threshold T. This may be most readily implemented using existing ASYSTD models and techniques. The only requirement is that the system delay time be previously determined so that a comparison may be made between corresponding points in the input and output bit streams. Determination of delay time T requires a preliminary ASYSTD simulation run. Once the delay time has been so determined, the input and output bit streams may be input to a comparator (using an ASYSTD COMPARATOR model). The output of the comparison process is then the number of bit errors α. An estimate of the bit error probability P is then:

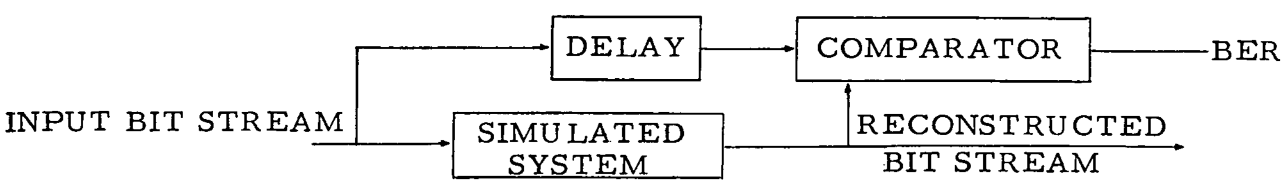
$$\hat{P} = \frac{\alpha}{K} \tag{39}$$

where K is the total number of bits during the period in which α errors are observed. The confidence levels associated with this estimate are discussed in the referenced report analysis.

An ASYSTD Bit Error Rate Meter suitable for performing BER estimates by direct error counting may be modeled as follows:

```
MODEL:  BER METER, ENDSIG, LAG, BT
INPUT <DELAY (LAG)> N1
N1 <COMP(ENDSIG, BT)> OUTPUT
END
```

The BER METER may be represented schematically as follows:



The DELAY model is already in the ASYSTD library. The COMPARATOR model is included in the Appendix.

SNR Estimation of Bit Error Rate

Of the other methods of estimating P_b (hence, BER) discussed in the Fourth Monthly Progress Report, the SNR method is useful in ASYSTD simulation. This method requires that the SNR first be estimated by the procedure discussed in Section 5.2. The SNR method makes use of the functional relationship between the baseband SNR and the bit error probability P_b (known for most operating systems). The SNR estimate can be made directly from the analog baseband bit signal at the decoder input. This method has the advantage of not requiring knowledge of the transmitted bit, although in a test situation this would be known a priori. In either instance, knowledge of sample statistics is required in order to relate the SNR and P_b. These relationships are best known for the case of Gaussian baseband signal statistics. Further considerations involved in SNR estimation of BER including confidence levels as a function of the number of observed bits, are detailed in the referenced discussion.

5.4 SIGNAL DISTORTION AND ERROR DETERMINATION FROM ASYSTD SIMULATIONS

Signal distortion components at each system node may be determined by appropriate post-processing of the ASYSTD simulated signal at that node. The choice of the appropriate post-processing routine to be used depends on the characteristics of the input signal and on the nature of the distortion to be determined.

For sinusoidal input signals, the output (or node) signal distortion may be characterized by percent total harmonic distortion (THD). The ASYSTD time domain representation of the signal at the node of interest must first be transformed to the frequency domain using a library fast fourier transform routine. The percent THD is then given by:

$$\text{THD} = \frac{100}{A}\sqrt{\sum_{\iota=1}^{N} D_\iota^2} \tag{40}$$

where:

D_i = Amplitude of the ith harmonic

A = Amplitude of the fundamental component (41)

N = Number of significant spectral components

Distortion of non-sinusoidal input signals may be examined in a similar manner if these are first resolved into their harmonic components. Each such component may then be processed separately by ASYSTD and the THD of the output (or node) signal due to that component determined as indicated above. For large numbers of components, however, total run time may prove prohibitive.

Although error criteria for non-sinusoidal and/or aperiodic analog signals are not uniquely defined, the following criteria are commonly employed:

1) Instantaneous Error, defined by:

$$e(t) = f(t) - g(t-T_o) \tag{42}$$

where:

$f(t)$ = input signal function (43)

$g(t-T_o)$ = output signal function translated back in time by the zero frequency group delay, T_o.

2) Instantaneous Error e(t) may be expressed as a percentage of the full scale input signal level.

3) Instantaneous Error e(t) may be expressed as a percentage of the instantaneous input signal level.

4) Mean Square Error, defined by:

$$e_{MS} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left[f(t) - g(t-T_o) \right]^2 dt \tag{44}$$

For transient, aperiodic signals, the Integral Square Error defined by:

$$e_I = \int_{\infty}^{\infty} \left[f(t) - g(t-T_o) \right]^2 dt \qquad (45)$$

is more applicable. Here the actual limits of integration may be limited to the duration of the signal, f(t).

Another error criterion is the Message Contrast Ratio, defined by:

$$E_{MCR} = \frac{\lim\limits_{T_1 \to \infty} \frac{1}{2T_1} \int_{-T_1}^{T_1} f^2(t)dt}{\lim\limits_{T_2 \to \infty} \frac{1}{2T_2} \int_{-T_2}^{T_2} \left[f(t) - g(t-T_o) \right]^2 dt} \qquad (46)$$

It has been assumed in the above definitions that the time delay is a constant T_o. That is, if T_o is the group delay for any frequency in the signal baseband of f(t):

$$t_D = T_o \Big|_{\omega \leq \omega_c} \qquad (47)$$

Since:

$t_D = \frac{\Delta\omega}{\Delta\theta}$, where θ is the phase, the assumption of constant To implies that the phase characteristic (θ vs ω) is a linear relationship.

Figure 5-3 illustrates the general dependence of time delay on phase linearity. As an indication of the actual error resulting from the assumption of constant group delay, Figure 5-4 shows group delay vs frequency for the particular cases of 6th order Bessel and Butterworth filters.

An ASYSTD post-processing routine (TEAP*) has been developed which evaluates the above criteria. Input data consists of an input signal array f(t), an output (or node) signal array g(t), a corresponding time array, and the value of the group delay, To. The last may be determined

* Transient Error Analysis Program

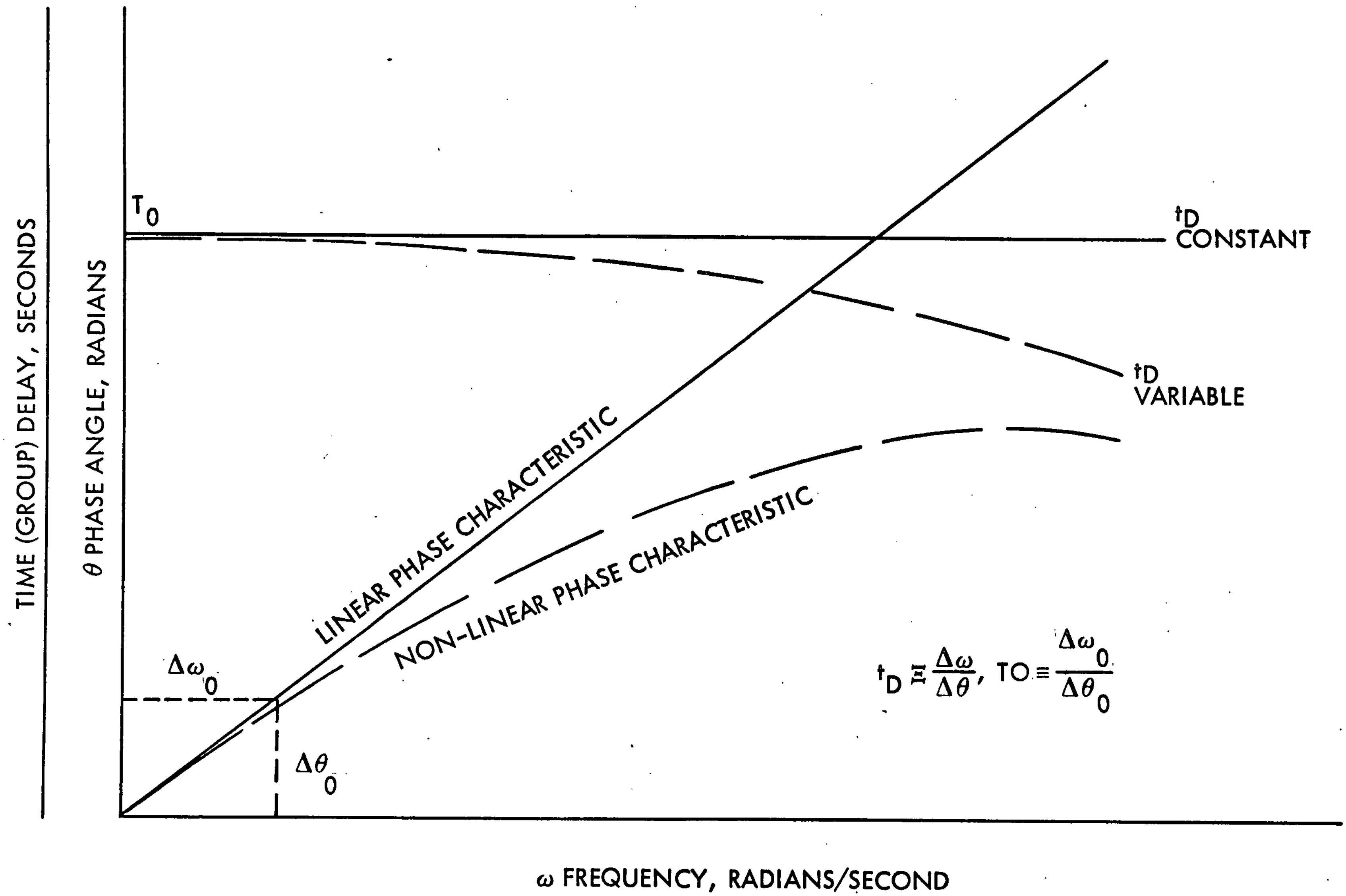


Figure 5-3. Example of Dependence of Time Delay on Slope of Phase vs. Frequency Characteristic

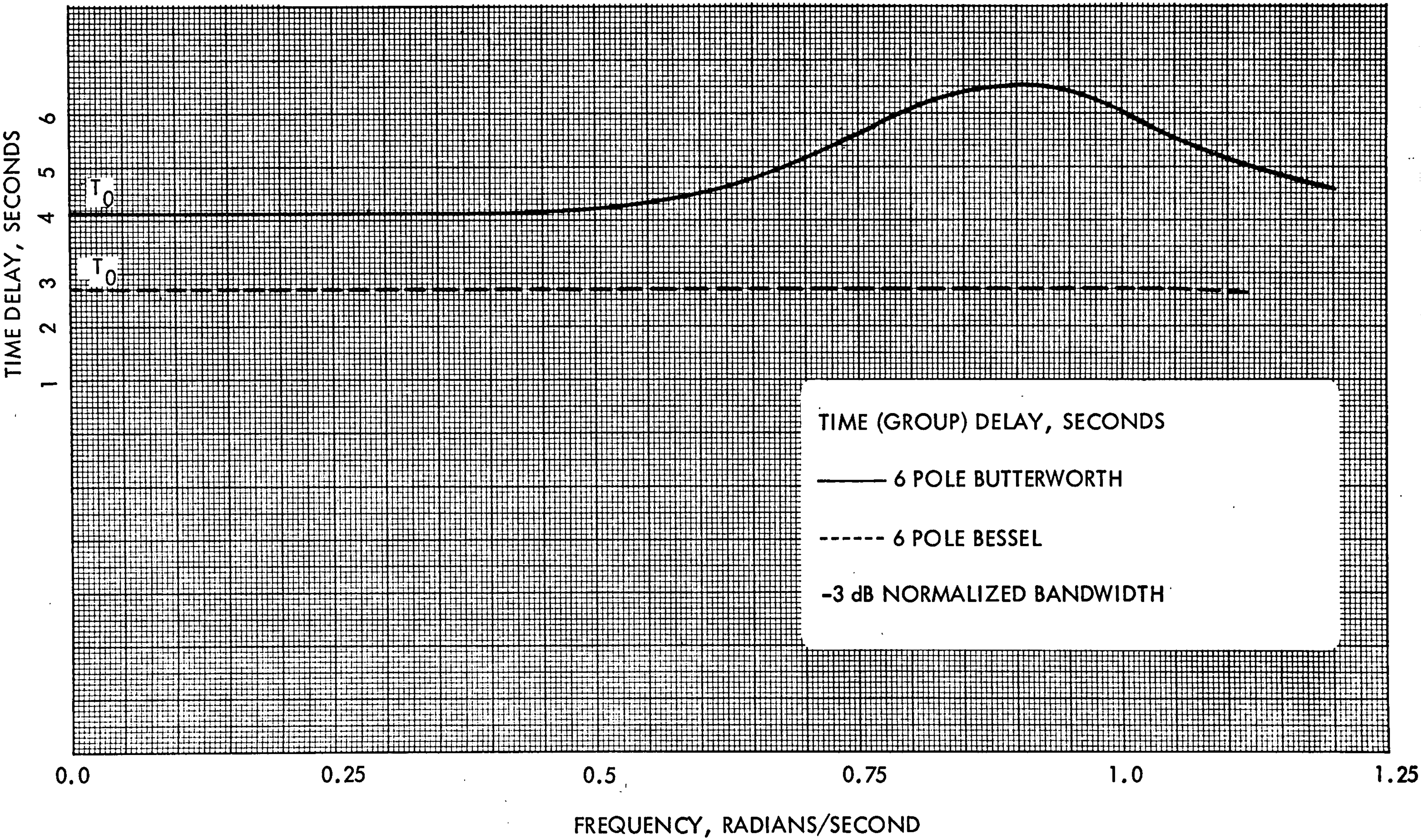


Figure 5-4. Normalized Time (Group) Delay Characteristic

from the phase response characteristic of the system. Depending on the nonlinearity of the system phase response, either the zero frequency group delay T_0 or a more suitable approximate value may be used. Typical results are indicated in Figures 5-5 and 5-6 for a Butterworth filter. Figure 5-5 depicts output signal error as a percent of the instantaneous input signal level. Figure 5-6 depicts output signal error as a percent of peak-to-peak input signal level and also indicates the corresponding calculated values of Mean Square Error, Integral Square Error, and Message Contrast Ratio.

Some problems, however, could lead to unacceptable computer run times since each such iteration requires an ASYSTD simulation of at least that portion of the system which was affected by the varied parameters. The practicality of such iterative techniques using ASYSTD is somewhat subjective and depends on:

1) Complexity of the system being simulated
2) Accuracy of the results desired
3) System optimization criterion being monitored
4) System parameters being optimized
5) Allowable computer time expenditures

Both the Direct Search and the Gradient methods have been implemented in readily available routines.

The adaptation of these existing optimization routines to an ASYSTD optimization procedure is conceptually straightforward. This task consists of creating a routine which functions as an interface between ASYSTD and the chosen optimization routine. The purpose of this required interface routine is twofold:

1) To execute an ASYSTD simulation given each new set of parameter values generated by the optimization routine
2) To calculate the chosen system optimization criterion by post-processing the ASYSTD simulation output

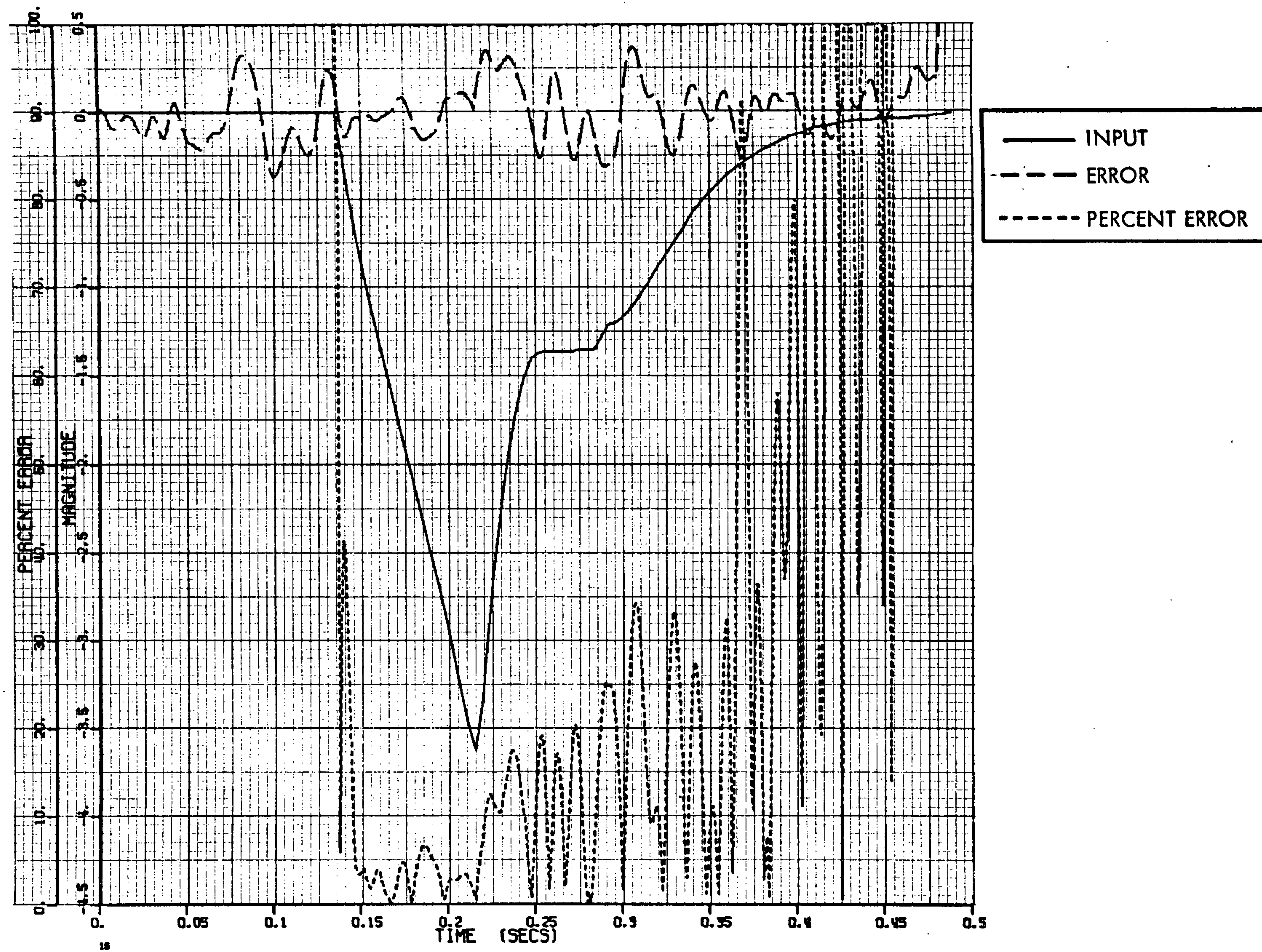


Figure 5-5. TEAP Generated Error and Percent Error: Butterworth Filter with 80 Hz Sinusoidal Input and 5-Percent Random Noise

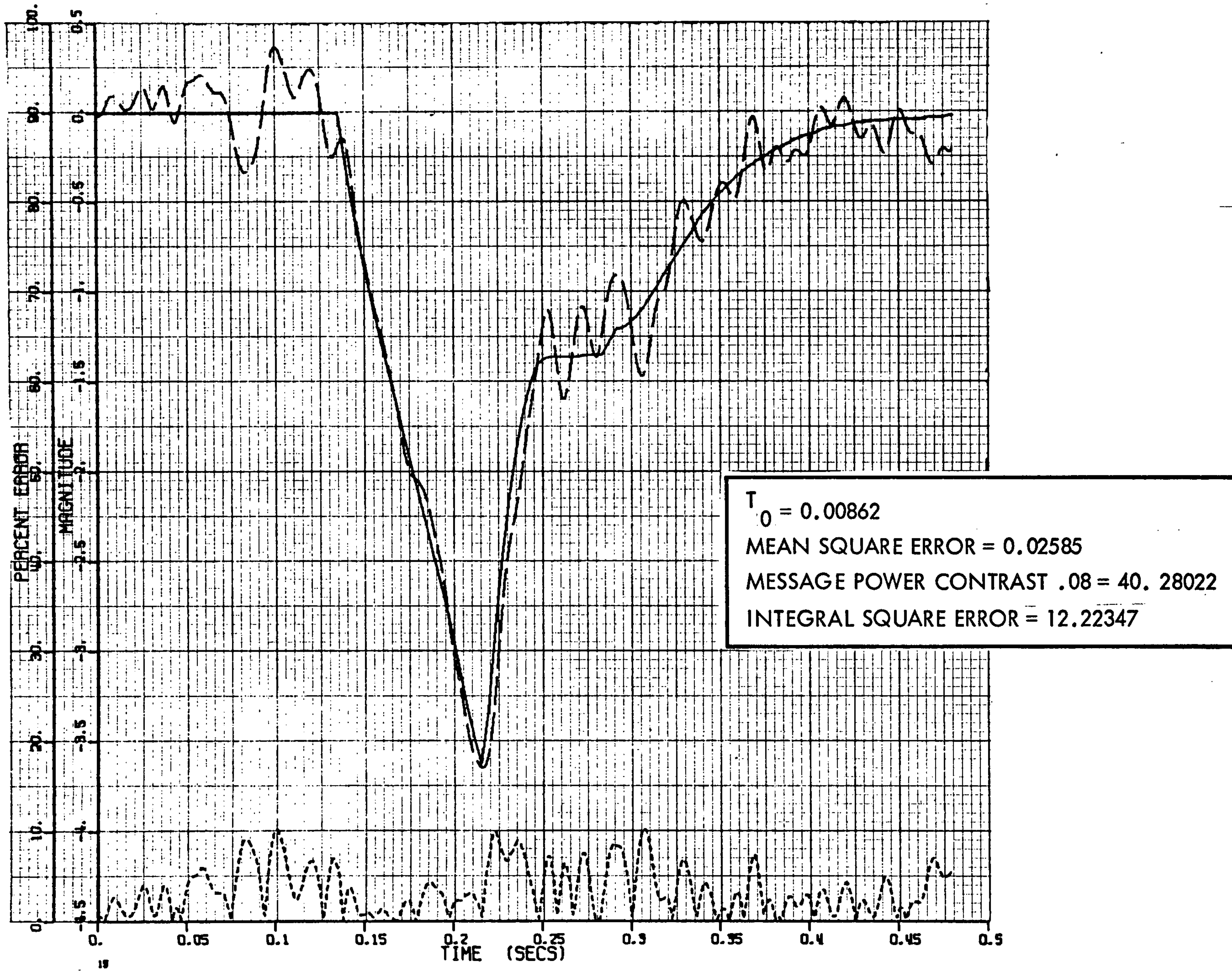


Figure 5-6. TEAP Generated Error Criteria: Butterworth Filter with 80 Hz Sinusoidal Input and 5-Percent Random Noise

5.5 FEASIBILITY OF SYSTEM OPTIMIZATION USING ASYSTD SIMULATION

Optimization, in its precise technical sense, involves maximizing (or minimizing) some system quality function of interest. The general optimization problem has three aspects:

1) Selection as an optimization criterion the most appropriate system performance index

2) Devising a suitable method for evaluating the selected criterion (i.e., an analysis routine)

3) Devising a procedure for maximizing (or minimizing) the chosen criterion which can be implemented with reasonable effort (i.e., an optimization routine)

In our case, the system optimization criterion is evaluated by post-processing the output signal of an ASYSTD simulation. The system optimization criterion may accordingly be chosen from any of the system performance functions which may be evaluated through a SYSTID simulation. Possible optimization criteria include signal to noise ratio (Section 5.0) and the various measures of signal distortion (Section 5.0) discussed previously. What remains then is the choice of a suitable optimization routine for systematically varying parameters of interest. The primary purpose of the optimization routine is to provide the analysis routine with a new and more optimum set of parameters. Figure 5-7 illustrates the relationship between the analysis and optimization routines. After each run, the results are examined and the parameters reevaluated for the next iteration. Hopefully through this iterative process the chosen system optimization criterion converges to an optimum value.

Two widely used optimization methods are the Direct Search and the Gradient (or steepest-descent) techniques. Direct Search is basically a trial-and-error technique guided by programmed logic which chooses successive values of the variable parameters, based upon the results obtained of the previous values. The Gradient method determines the choice of parameter values based on the "derivative" of the optimization criterion with respect to each variable parameter. Both methods may be applicable.

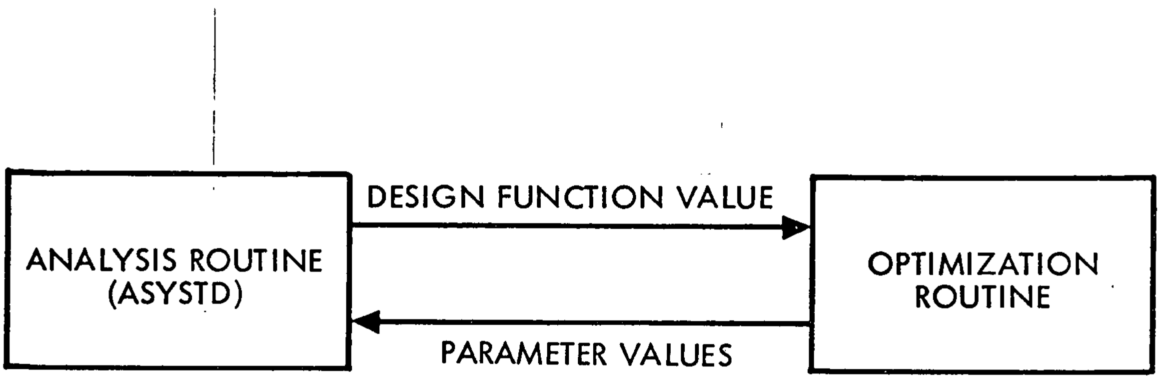


Figure 5-7. Relationship of Analysis and Optimization Routines.

Some logical procedures must be incorporated so that only that portion of the simulation actually affected by the previous parameter revisions is repeated in the following iteration. That is, the signal prior to the first modified element in the system is retained from the previous iteration rather than regenerated each time. Furthermore, in the interest of reduced run time when optimizing more complex systems, it may prove practicable to truncate the system by examining the output at some intermediate critical stage.

APPENDIX A

COMPARATOR MODEL

```
      SUBROUTINE COMP(ENDSIG, BT)
      INCLUDE HEDFOR, LIST
      DIMENSION I(2)
      EQUIVALENCE (I, V)
      DEFINE ACCUM = I(Z+1)
      DEFINE ERROR  = I(Z+2)
      DEFINE ICOUNT = I(Z+3)
      Z    = ZZ
      ZZ   = ZZ+3
      ICOUNT  =  ICOUNT+1
      IF (ICOUNT.GE.INT (BT/DT + .5))  ICOUNT  =  0
      IF (ICOUNT.NE.O)  GO TO 1O
      ACCUM  =  ACCUM+1
      IF (ABS (ENDSIG - V(VIN)).GT..5)  ERROR  =  ERROR+1
1O    IF (ACCUM.EQ.O)  RETURN
      V(VOUT)  =  ERROR/ACCUM
      RETURN
      END
```

APPENDIX B

REFERENCES

(1) Communication Satellite Corporation, "Orthogonal Transform Feasibility Study," Monthly Progress Reports, October 1 through March 1, 1971, for NASA under Contract No. NAS9-11240

(2) Systems Associates, Inc., "Advanced Communication System Study," Monthly Progress Report No. 1, pp 5-21, June 1971, for NASA under Contract No. NAS9-11743

(3) ASYSTD Fourth Monthly Progress Report, September 28, 1971, Contract No. NAS9-11743, "Advanced Communication System Time Domain Modeling Techniques Study"